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Rare K Decays *

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The rare decays of the K meson have had a long tradition as a laboratory for testing the symmetry properties of the weak interactions, and the manner in which these symmetries are broken by higher order effects. Present–day interest is focussed on decays that are suppressed by CP–symmetry or GIM symmetry. Such decays, in the standard theory, are sensitive to effects of the virtual top quark, and could also reveal new interactions transcending the standard model. In addition, the radiative decays of the K meson have become a useful testing–ground for effective Lagrangians describing the low energy interactions of pions, kaons and photons.

This talk is a selective review of some rare K processes. For a more comprehensive discussion, we refer to the reviews listed in [1].

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1 Charged Current Rarities

An example of a decay that is rare, yet allowed in the lowest order, is the $\Delta S = 0$ transition $K^0 \to K^+ e^- \bar{\nu}_e$ (Fig. 1a). This is closely analogous to pion β-decay, $\pi^- \to \pi^0 e^- \bar{\nu}_e$. Conservation of the vector current $\bar{u}\gamma_\mu d$ dictates that the relevant matrix element is $\langle K^0|I_-|K^+\rangle = 1$ (in analogy to $\langle \pi^0|I_-|\pi^+\rangle = \sqrt{2}$) and the predicted branching ratio is

$$B(K_L \to K^+ e^- \nu_e) = 3 \times 10^{-9}. \tag{1}$$

A curious analogue of the $\Delta S = \Delta Q$ decay $K^+ \to \pi^0 e^+ \nu_e$ is the hypothetical transition $K^+ \to \text{"}\eta\text{"} e^+ \nu_e$. The latter is not observable as a real decay process, since $m_\eta > m_K$. The matrix elements for $K^+ \to \eta$ and $K^+ \to \pi^0$ are $-1/\sqrt{6}$ and $1/\sqrt{2}$, respectively. A possible way to probe the $K^+ \to \eta$ coupling is via the decay

$$K^+ \to \gamma\gamma e^+ \nu_e. \tag{2}$$

Some of the relevant diagrams are shown in Fig. 1b. An estimate of this decay would be of interest.

In the second order of weak interactions, it is possible to obtain a $\Delta S = -\Delta Q$ transition $K^0 \to \pi^+ e^- \bar{\nu}_e$ (Fig. 1c). A model based on the parity–conserving vertices $K^0 - \pi^0$ and $K^+ - \pi^+$, combined with the $\Delta S = \Delta Q$ transitions $\pi^0 \to \pi^+ l^- \bar{\nu}_l$ and $K^0 \to K^+ l^- \bar{\nu}_l$ gives [2]

$$\frac{\Gamma(\Delta S = -\Delta Q)}{\Gamma(\Delta S = +\Delta Q)} = 0.5 \times 10^{-12} \qquad (K_{e3}, K_{\mu3}). \tag{3}$$

An alternative mechanism, combining the parity–violating decay $K^0 \to \pi^+ \pi^-$ with the transition $\pi^- \to \mu^- \bar{\nu}_\mu$, yields [3]

$$\frac{\Gamma(\Delta S = -\Delta Q)}{\Gamma(\Delta S = +\Delta Q)} = 1.1 \times 10^{-12} \qquad (K_{\mu3} \text{ only}). \tag{4}$$

This second mechanism produces a $\Delta S = -\Delta Q$ amplitude $K^0 \to \pi^+ \mu^- \nu_\mu$ with a helicity structure and pion energy spectrum that is quite different from that in the $\Delta S = \Delta Q$ transition $\bar{K}^0 \to \pi^+ \mu^- \bar{\nu}_\mu$. To the extent that both mechanisms are possible, the e/μ ratio in $\Delta S = -\Delta Q$ transitions will be different from that in $\Delta S = \Delta Q$ decays.

2 Neutral Current Decay into $l^+ l^-$ Pairs

2.1 Decay $K_L \rightarrow \mu^+\mu^-$

The decay $K_L \rightarrow \mu^+\mu^-$ has a well–known unitarity bound [4]

$$\frac{\Gamma(K_L \rightarrow \mu^+\mu^-)}{\Gamma(K_L \rightarrow 2\gamma)} \geq \frac{\alpha^2}{2\beta} \frac{m_\mu^2}{m_K^2} \left(\ln \frac{1+\beta}{1-\beta}\right)^2, \qquad \beta = \sqrt{1 - 4m_\mu^2/m_K^2} \quad (5)$$

associated with the 2γ intermediate state (Fig. 2a). The corresponding branching ratio is

$$B_{abs} = (6.8 \pm 0.3) \times 10^{-9}. \tag{6}$$

If one includes the dispersive part of the 2γ amplitude, as well as the real amplitude induced by the short–distance interaction $s\bar{d} \rightarrow \mu^+\mu^-$ (box and penguin diagrams, Fig. 2b), the full branching ratio is

$$B(K_L \rightarrow \mu^+\mu^-) = B_{abs} + |\sqrt{B_{disp}} \pm \sqrt{B_{short-dist.}}|^2. \tag{7}$$

A theoretical estimate of B_{disp} requires a model for the $K_L \rightarrow \gamma^*\gamma^*$ form factor. Vector–meson–dominance models [5], similar to those used for $\pi^0, \eta \rightarrow \gamma^*\gamma^*$, tend to give $B_{disp}/B_{abs} \leq 0.1$ (to be compared with the experimental rate of $\eta \rightarrow \mu^+\mu^-$, $B(\eta \rightarrow \mu^+\mu^-) = (1.3 \pm 0.1) \times B_{abs}(\eta \rightarrow \mu^+\mu^-)$). A model for $K_L \rightarrow \gamma^*\gamma$ [6], that describes the observed spectrum of Dalitz pairs in $K_L \rightarrow \gamma e^+ e^-$ [7], yields an even smaller value of B_{disp}.

The short–distance branching ratio has been calculated to be [8, 9]

$$B_{short-dist.}(K_L \rightarrow \mu^+\mu^-) = 6.4 \times 10^{-3}$$
$$\cdot \left[P_0(K_L \rightarrow 2\mu) + \mathrm{Re}\lambda_t Y(x_t)\right]^2 \tag{8}$$

where

$$Y(x_t) = \frac{x_t}{8}\left[\frac{x_t - 4}{x_t - 1} + \frac{3x_t}{(x_t - 1)^2} \ln x_t\right], \qquad x_t = \frac{m_t^2}{m_W^2}, \tag{9}$$

and

$$\lambda_t = V_{ts}^* V_{td} = -A^2\lambda^5(1 - \rho - i\eta). \tag{10}$$

The term $P_0(K_L \rightarrow 2\mu)$ denotes a residual charm quark contribution, estimated to be -0.75×10^{-4}, for $m_c = 1.4$ GeV, $\Lambda = 200$ MeV.

The experimental branching ratio $B_{exp}(K_L \rightarrow \mu^+\mu^-) = (7.4 \pm 0.4) \times 10^{-9}$ [10], is only slightly in excess of the unitarity bound (Eq. (6)), and can constrain the parameter $\mathrm{Re}\lambda_t = -A^2\lambda^5(1 - \rho)$ appearing in the short–distance contribution. At present, the uncertainty in the 2γ dispersive rate B_{disp} limits the efficacy of this decay in providing a stringent constraint on ρ [11].

2.2 Decay $K_L \to \pi^0 l^+ l^-$

In the one–photon exchange approximation, CP–symmetry forbids the decay $K_2 \to \pi^0 l^+ l^-$ but allows the decay $K_1 \to \pi^0 l^+ l^-$ [12]. A CP–conserving amplitude for $K_2 \to \pi^0 l^+ l^-$ is possible through a 2γ intermediate state (Fig. 2a). The 2γ–induced amplitude is proportional to m_l if the two photons have $J = 0$; however, dynamical models of $K_2 \to \pi^0 \gamma\gamma$ permit also a $J = 2$ component which yields an amplitude for $K_2 \to \pi^0 l^+ l^-$ unsuppressed by a factor m_l [13]. It follows that the amplitude for $K_L \to \pi^0 l^+ l^-$ will contain three components

$$A = \alpha^2 A_{2\gamma} + \alpha\varepsilon A_{1\gamma} + \eta\lambda^4 A_{sd}, \tag{11}$$

where the second and third pieces denote amplitudes associated with indirect and direct (short–distance) CP–violation. The factors α^2, $\varepsilon\alpha$ and $\eta\lambda^4$ express the orders of magnitude of these components, and underscore the fact that they are, a priori, comparable in size.

The CP–conserving amplitude for $K_2 \to \pi^0 l^+ l^-$ involving a 2γ intermediate state has the form [14]

$$A(K_2(p) \to \pi^0(p') + l^-(k) + l^+(k'))_{2\gamma} = -\frac{\alpha}{16}$$
$$\cdot \bar{u}(k)\Big[\frac{G_{eff}}{M_\rho^2}F_1 q\!\!\!/ + F_2\Big]v(k'), \tag{12}$$

where F_2 is proportional to m_l, and is negligible for $K_2 \to \pi^0 e^+ e^-$. The coefficient G_{eff}/M_ρ^2 is an effective coupling constant appearing in the $K_2 \to \pi^0 \gamma\gamma$ amplitude, parametrized as

$$\begin{aligned}
A(K_2(p) \to \pi^0(p')\gamma(q)\gamma'(q')) &= 2\frac{G_{eff}}{M_\rho^2}\Big[(\epsilon \cdot \epsilon')(q \cdot p)(q' \cdot p) \\
&+ (q \cdot q')(\epsilon \cdot p)(\epsilon' \cdot p) - (\epsilon \cdot p)(\epsilon' \cdot q)(q' \cdot p) \\
&- (\epsilon \cdot q')(q \cdot p)(\epsilon' \cdot p)\Big] + \cdots
\end{aligned} \tag{13}$$

where the ellipsis denote terms that are ineffective for $K_L \to \pi^0 e^+ e^-$. An analysis of the data on $K_L \to \pi^0 \gamma\gamma$ [15] suggests $G_{eff} \sim 0.15 \times 10^{-7} m_K^{-2}$ [14]. The form factor F_1 has an absorptive part

$$\begin{aligned}
\mathrm{Im}F_1 &= -\frac{\Delta}{\beta^2}\Big[\frac{2}{3} + \frac{2}{\beta^2} - (\frac{1}{\beta^2} - \beta^2)\frac{1}{\beta}\ln\frac{1+\beta}{1-\beta}\Big] \\
&\overset{\beta\to 1}{\to} \frac{8}{3}\Delta,
\end{aligned} \tag{14}$$

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where $\Delta = -2p \cdot (k - k')$, $\beta = \sqrt{1 - 4m_l^2/s}$, $s = (k + k')^2$. The corresponding CP–conserving branching ratio is

$$B_{CPC}(K_L \rightarrow \pi^0 e^+ e^-) = 1.7 \times 10^{-12} \left(\frac{G_{eff} m_K^2}{0.15 \times 10^{-7}}\right)^2 (1 + \rho_{disp}), \quad (15)$$

ρ_{disp} being the dispersive part, estimated in [14] to be 1.5.

The result (15) has to be compared with the CP–violating rate [16]

$$B_{CPV}(K_L \rightarrow \pi^0 e^+ e^-) = \Big[|0.76 r e^{i\pi/4} + i\tilde{C}_V \frac{\eta}{0.4}|^2$$
$$+ |\tilde{C}_A \frac{\eta}{0.4}|^2\Big] \times 10^{-11}, \quad (16)$$

where $\tilde{C}_V$ and $\tilde{C}_A$ are coupling constants describing the short–distance interaction $s\bar{d} \rightarrow e^+ e^-$, with numerical values $\tilde{C}_V = -0.6$, $\tilde{C}_A = 0.7$ for $m_t = 170$ GeV. The parameter r, characterising the indirect CP–violating amplitude, is defined as

$$r = \Big[\frac{\Gamma(K_1 \rightarrow \pi^0 e^+ e^-)}{\Gamma(K^+ \rightarrow \pi^+ e^+ e^-)}\Big]^{1/2}. \quad (17)$$

For $\eta = 0.4$, the CP–violating branching ratio B_{CPV} is $1.2 \times 10^{-11} (2.1 \times 10^{-11})$ for $r = +1(-1)$. Models with much larger and much smaller values of r are possible. These estimates suggest that the CP–violating rate B_{CPV} is somewhat higher than the CP–conserving rate B_{CPC}. A precise measurement of the rate and spectrum of $K_2 \rightarrow \pi^0 \gamma\gamma$ would help to sharpen the estimate of B_{CPC}, while a measurement of r is needed to predict B_{CPV}. Expectations for the decay $K_L \rightarrow \pi^0 \mu^+ \mu^-$, as well as an estimate of the CP–violating $l^+ l^-$ energy asymmetry are given in [14]. Present experimental limits (from the E799 experiment at Fermilab) are $B(K_L \rightarrow \pi^0 e^+ e^-) < 4.3 \times 10^{-9}$, $B(K_L \rightarrow \pi^0 \mu^+ \mu^-) < 5.1 \times 10^{-9}$ [17].

2.3 Decay $K_L \rightarrow \pi^+ \pi^- e^+ e^-$

A study of the photon spectrum in the decay $K_L \rightarrow \pi^+ \pi^- \gamma$ shows two clear components [18]: (i) bremsstrahlung from the CP–violating decay $K_L \rightarrow \pi^+ \pi^-$, and (ii) direct photon emission of magnetic dipole nature from the CP–conserving decay $K_2 \rightarrow \pi^+ \pi^- \gamma$. The simultaneous presence of bremsstrahlung and $M1$ amplitudes implies that the photon in the decay $K_L \rightarrow \pi^+ \pi^- \gamma$ has a CP–violating circular polarization.

The conversion process $K_L \to \pi^+\pi^- e^+ e^-$ may be viewed as a means of probing this polarization, by studying the correlation of the $e^+ e^-$ plane relative to the $\pi^+\pi^-$ plane.

An analysis based on the amplitude

$$A(K_L \to \pi^+(p_+)\pi^-(p_-)e^+(k_+)e^-(k_-)) = e|f_S|\Big[g_{Br}(\frac{p_+^\mu}{p_+\cdot k} - \frac{p_-^\mu}{p_-\cdot k})$$

$$+ g_{M1}\varepsilon_{\mu\nu\rho\sigma}k^\nu p_+^\rho p_-^\sigma\Big]\frac{e}{k^2}\bar{u}(k_-)\gamma_\mu v(k_+)(18)$$

was carried out in [19]. Here $k = k_+ + k_-$, f_S is the amplitude of $K_S \to \pi^+\pi^-$, and g_{Br} and g_{M1} are given empirically by $g_{Br} = \eta_{+-}e^{i\delta_0(m_K^2)}$, $g_{M1} = i(0.76)e^{i\delta_1(s_\pi)}$, $\delta_{0,1}$ being the s– and p–wave $\pi\pi$ phase shifts. A significant CP–violating asymmetry was found in the Φ–distribution of the process, Φ being the angle between the e^+e^- and $\pi^+\pi^-$ planes:

$$
\begin{aligned}
A &= \frac{\displaystyle\int_0^{\pi/2} d\Phi \frac{d\Gamma}{d\Phi} - \int_{\pi/2}^{\pi} d\Phi \frac{d\Gamma}{d\Phi}}{\displaystyle\int_0^{\pi} d\Phi \frac{d\Gamma}{d\Phi}} \\
&= 15\% \sin(\Phi_{+-} + \delta_0(m_K^2) - \bar{\delta}_1) \\
&\approx 14\%,
\end{aligned}
\tag{19}
$$

where $\bar{\delta}_1$ denotes an average p–wave phase ($\sim 10^0$). This analysis was extended in [20] to include short–distance CP–violation, contained in the effective Hamiltonian for $s\bar{d} \to e^+e^-$. These direct CP–violating effects were found to be very small ($< 10^{-3}$). The branching ratio of $K_L \to \pi^+\pi^- e^+ e^-$ is predicted to be 3×10^{-7}, so that the large asymmetry given in Eq. (19) may well be accessible in the next round of experiments.

2.4 Decay $K^+ \to \pi^+ l^+ l^-$

The decays $K^+ \to \pi^+ l^+ l^-$ are dominated by one–photon exchange, and are principally of interest as tests of the $K\pi\gamma^*$ vertex. In chiral pertubation theory, this vertex is calculable in terms of the effective Lagrangian $\mathcal{L}_{eff}(\pi, K, \gamma)$, and the matrix element has the form [21]

$$A(K^+(k) \to \pi(p) + l^+ + l^-) = \frac{\alpha G_8}{4\pi}C^+(z)\bar{u}(\not{k} + \not{p})v,$$

$$z = (k-p)^2/m_K^2, \tag{20}$$

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where $G_8 = G_F/\sqrt{2}V_{ud}V_{us}^* g_8$, $g_8 = 5.1$. The function $C^+(z)$ is known, up to an additive constant w_+. In terms of this constant, the branching ratios are calculated to be

$$
\begin{aligned}
B(K^+ \to \pi^+ e^+ e^-) &= (3.15 - 21.1w_+ + 36.1w_+^2) \times 10^{-8}, \\
B(K^+ \to \pi^+ \mu^+ \mu^-) &= (3.93 - 32.7w_+ + 70.5w_+^2) \times 10^{-9}. \quad (21)
\end{aligned}
$$

The recent measurement [22] $B(K^+ \to \pi^+ e^+ e^-) = (2.99 \pm 0.22) \times 10^{-7}$ implies $w_+ = 0.89^{+0.26}_{-0.14}$, and leads to the prediction $B(K^+ \to \pi^+ \mu^+ \mu^-) = 3.07 \times 10^{-8}$.

Refinements to the matrix element (20) occur if one takes into account the short–distance interaction $s\bar{d} \to l^+ l^-$ [23]. One interesting aspect of these corrections is the addition to the matrix element (20) of a parity–violating term of the form

$$
A_{sd}(K^+(k) \to \pi(p) + l^+ + l^-) = \frac{G_F \alpha}{\sqrt{2}} V_{us}\Big[B(k+p)^\mu + C(k-p)^\mu\Big]
$$
$$
\bar{u}\gamma_\mu \gamma_5 v. \,(22)
$$

The short–distance interaction gives $B = f_+\xi$, $C = f_-\xi$, where

$$
\xi = -1.4 \times 10^{-4} - \frac{Y(x_t)}{2\pi \sin^2 \theta_W} A^2 \lambda^4 (1 - \rho - i\eta), \quad (23)
$$

f_+, f_- being the two form factors of K_{l3} decay ($f_+ \approx 1$, $f_- \approx 0$). Interference with the leading one–photon exchange amplitude (20) gives rise to a parity–violating longitudinal polarization of the μ^+ [23]

$$
|A_{LR}| \equiv |\frac{\Gamma_L - \Gamma_R}{\Gamma_L + \Gamma_R}| = |2.3\mathrm{Re}\xi|, \quad (24)
$$

thus providing a way to determine the parameter ρ.

Another consequence [24] of the short–distance amplitude (22) (which we write as $F_A \bar{u}(\slashed{p} + \slashed{k})\gamma_5 v$) is that its interference with the dominant one–photon amplitude (20) (which we write as $F_V \bar{u}(\slashed{p}+\slashed{k})v$) produces a T–odd, P–odd term in the decay rate, of the form $\mathrm{Im}(F_V F_A^*)(\vec{s}_+ \times \vec{s}_-)\cdot\vec{p}$, where $\vec{s}_+$ and $\vec{s}_-$ are spin vectors of the μ^+ and μ^- in the K rest frame, and $\vec{p}$ is the μ^+ momentum. Such a term can probe the CP-violating parameter η of the short–distance interaction. Detection of this term, however, requires measurement of a correlation between the spins of both μ^+ and μ^-, a difficult task.

An important challenge for the low energy effective Lagrangian

$\mathcal{L}_{eff}(K, \pi, \gamma)$ is to produce a reliable prediction for the decay $K_1 \rightarrow \pi^0 l^+ l^-$. In chiral pertubation theory, the matrix element is determined up to an unknown constant w_S [21]. As discussed in Section 2.2, information on this decay mode is important for determining the magnitude of indirect CP–violation in $K_L \rightarrow \pi^0 l^+ l^-$.

3   Neutral Current Decays into $\nu\bar\nu$ Pairs

3.1   Decay $K^+ \rightarrow \pi^+\nu\bar\nu$

The decay $K^+ \rightarrow \pi^+\nu\bar\nu$ is a short–distance dominated reaction, determined by the box and penguin graphs shown in Fig. 3a. The branching ratio is predicted to be [25]

$$B(K^+ \rightarrow \pi^+\nu_l\bar\nu_l) = 5.9 \times 10^{-5} |P_0(K^+ \rightarrow \pi^+\nu_l\bar\nu_l) + \lambda_t X(x_t)|^2, \quad (25)$$

where

$$P_0(K^+ \rightarrow \pi^+\nu_l\bar\nu_l) = \begin{cases} -2.5 \times 10^{-4} & \text{for} & \nu_e, \nu_\mu \\ -1.7 \times 10^{-4} & \text{for} & \nu_\tau \end{cases} \quad (26)$$

and

$$X(x_t) = \frac{x_t}{8} \Big[\frac{x_t + 2}{x_t - 1} + \frac{3x_t - 6}{(x_t - 1)^2} \ln x_t \Big]. \quad (27)$$

Summing over all three neutrino flavours, one obtains a typical value $B(K^+ \rightarrow \pi^+\nu\bar\nu) = 1.3 \times 10^{-10}$ for $\rho = 0$, $\eta = 0.4$, with a possible range $(0.5 - 5) \times 10^{-10}$ for the presently allowed domain of (ρ, η). The present experimental limit is 5.2×10^{-9} (AGS E787) [26] and a sensitivity of 10^{-10}/event is within reach.

The long–distance contributions to $K^+ \rightarrow \pi^+\nu\bar\nu$, symbolised by the diagrams in Fig. 3b, were calculated in [27]. In particular, the hadronic contribution to the $K^+\pi^+Z$ vertex was obtained using current algebra arguments. It was concluded that these effects are three orders of magnitude smaller than the short–distance contribution. A more recent calculation [28], using chiral pertubation theory, has found a very similar result. (For additional remarks, see [29]).

3.2   Decay $K_L \rightarrow \pi^0\nu\bar\nu$

Finally, an example of a short–distance dominated process, which is at the same time purely CP–violating, is the decay $K_2 \rightarrow \pi^0\nu\bar\nu$ [30]. Its

branching ratio is given by [31]

$$
\begin{aligned}
B(K_L \to \pi^0 \nu \bar{\nu}) &= 7.32 \times 10^{-4} \big[\mathrm{Im}\lambda_t\big]^2 X^2(x_t) \\
&= 1.94 \times 10^{-10} \big[\eta^2 A^4\big] X^2(x_t). \tag{28}
\end{aligned}
$$

While experimentally remote (the present limit is $B(K_L \to \pi^0 \nu \bar{\nu}) < 5.8 \times 10^{-5}$ [32]), this reaction is an interesting example of a process that directly measures the CP–violating paramter η, with essentially no hadronic uncertainties.

4 Radiative Decays

The radiative decays of the K mesons, such as $K_{L,S} \to \gamma\gamma$, $K_L \to \pi^0\gamma\gamma$, $K_L \to \pi^+\pi^-\gamma$, $K^+ \to \pi^+\pi^0\gamma$, $K^+ \to \pi^+\gamma\gamma$, $K_L \to \pi^0\pi^0\gamma\gamma$,$K_{L,S} \to \pi^0\pi^0\gamma$ etc. are a source of abundant grist for models, such as chiral pertubation theory [33], that attempt to describe the low energy interactions of pions, kaons and photons. The interplay of chiral symmetry, CP–symmetry and gauge invariance, combined with the weak nonleptonic $\Delta I = 1/2$ rule, produces interesting patterns and hierarchies amongst the various channels. We will limit our remarks here to two reactions,

$$
\begin{aligned}
K_{L,S} &\to \pi^0\pi^0\gamma, \\
K_{L,S} &\to 3\gamma \tag{29}
\end{aligned}
$$

that have the piquant feature of being *quadrupole* transitions.

4.1 Decay $K_{L,S} \to \pi^0\pi^0\gamma$

Gauge invariance implies that the $\pi^0\pi^0$ system in $K_{L,S} \to \pi^0\pi^0\gamma$ cannot have $J = 0$ (since that would amount to a $0 \to 0$ radiative transition). Bose statistics implies that the $\pi^0\pi^0$ pair cannot have $J = 1$ (since such a state would not be symmetric under exchange of the two π^0's). It follows that the $\pi^0\pi^0$ state has at least two units of angular momentum, and the associated photon corresponds to quadrupole radiation [34].

CP–invariance implies that the decays $K_L \to \pi^0\pi^0\gamma$ and $K_S \to \pi^0\pi^0\gamma$ are $E2$ and $M2$ transitions, respectively, with matrix elements

$$
A(K_L \to \pi^0(p_1)\pi^0(p_2)\gamma(k)) = \frac{g_{E2}}{m_K^3} \frac{(p_1 - p_2) \cdot k}{\Lambda^2} \Big[(\epsilon \cdot p_1)(k \cdot p_2)
$$

$$-(\epsilon \cdot p_2)(k \cdot p_1)\Big],$$

$$A(K_S \to \pi^0(p_1)\pi^0(p_2)\gamma(k)) = \frac{g_{M2}}{m_K^3} \frac{(p_1 - p_2) \cdot k}{\Lambda^2} \varepsilon_{\mu\nu\rho\sigma} \epsilon^\mu k^\nu p_1^\rho p_2^\sigma \quad (30)$$

A qualitative estimate of the decay rates may be obtained by making a comparison with the measured $M1$ transition $K_L \to \pi^+\pi^-\gamma$ [16], parametrized as

$$A(K_L \to \pi^+(p_+)\pi^-(p_-)\gamma(k)) = \frac{g_{M1}}{m_K^3} \varepsilon_{\mu\nu\rho\sigma} \epsilon^\mu k^\nu p_+^\rho p_-^\sigma. \quad (31)$$

Assuming that the dimensionless couplings g_{M1}, g_{M2} and g_{E2} are similar in magnitude, and that the reaction "radius" $1/\Lambda$ is of order $1/m_\rho$, we obtain [34]

$$\begin{aligned} B(K_L \to \pi^0\pi^0\gamma) &\approx 1.0 \times 10^{-8}, \\ B(K_S \to \pi^0\pi^0\gamma) &\approx 1.7 \times 10^{-11}. \end{aligned} \quad (32)$$

In chiral pertubation theory, the reaction $K_L \to \pi^0\pi^0\gamma$ occurs only in order p^6 in the chiral expansion [35]. On the other hand the reaction $K_L \to \pi^0\pi^0\gamma^*$, with a virtual photon, is possible in order p^4. It follows that a study of the Dalitz pair process $K_L \to \pi^0\pi^0 e^+e^-$ could show two components, one associated with $K_L \to \pi^0\pi^0\gamma$ ($E2$; $O(p^6)$), which is strongly peaked at low e^+e^- masses, and one associated with $K_L \to \pi^0\pi^0\gamma^*$ ($O(p^4)$) which shows up as a broad continuum [34].

4.2   Decay $K^0 \to$ Three Photons

Given that $B(K_L \to 2\gamma) = 5.7 \times 10^{-4}$, $B(K_S \to 2\gamma) = 2.4 \times 10^{-6}$ [36] it is interesting to ask what one expects for the decays $K_{L,S} \to 3\gamma$ [37].

First of all, it should be noted that both $K_L \to 3\gamma$ and $K_S \to 3\gamma$ are possible without violating CP or any other general symmetry principle. Gauge invariance dictates that no pair of photons in these channels can have $J = 0$, while Bose statistics forbids any pair from having $J = 1$ (Yang's theorem). It follows that every pair of photons in these decays must have at least two units of angular momentum. A simple model that relates the decays $K_{L,S} \to 3\gamma$ to the other quadrupole transition $K_{L,S} \to \pi^0\pi^0\gamma$ discussed above yields [37]

$$\begin{aligned} B(K_L \to 3\gamma) &= 3 \times 10^{-19}, \\ B(K_S \to 3\gamma) &= 5 \times 10^{-21}. \end{aligned} \quad (33)$$

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Thus the 3γ decay mode is suppressed relative to the 2γ mode by *15 orders of magnitude* – a remarkable reminder of the power of Bose statistics in this year of the Bose centenary!

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Figure Captions

Fig. 1. Diagrams relevant for the decays (a) $K^0 \to K^+ e^- \bar{\nu}_e$, (b) $K^+ \to \gamma\gamma e^+ \nu_e$, (c) $\Delta S = -\Delta Q$ decays $K^0 \to \pi^+ l^- \bar{\nu}_l$.

Fig. 2. (a) Two–photon contribution to $K_L \to l^+ l^-$ and $K_L \to \pi^0 l^+ l^-$. (b) Diagrams describing the short–distance interaction $s\bar{d} \to l^+ l^-$.

Fig. 3. (a) Short–distance diagrams relevant to the interaction $s\bar{d} \to \nu\bar{\nu}$. (b) Long–range contributions to the decay $K^+ \to \pi^+ \nu\bar{\nu}$.

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